Bayesian Networks with Applications in Reliability Analysis

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Outline

→ Motivation

→ Brief introduction to Bayesian networks

→ Summary of the papers:

   **Paper I — Langseth & Lindqvist (2002):**
   A maintenance model for components exposed to several failure modes and imperfect repair. *Tech. rep., Submitted as invited paper.*

   **Paper II — Langseth & Jensen (2002):**
   Decision theoretic troubleshooting of coherent systems. *RESS – Forthcoming.*

   **Paper III — Langseth & Nielsen (2002a):**
   Classification using hierarchical naïve Bayes models. *Tech. rep., Submitted to JMLR.*

   **Paper IV — Langseth & Bangsø (2001):**
   Parameter learning in object oriented Bayesian networks. *Annals of Math & AI.*

   **Paper V — Langseth & Nielsen (2002b):**
   Fusion of domain knowledge with data for structural learning in object oriented domains. *JMLR – Forthcoming.*
Motivation

Situation:
I am about to provide input to a decision problem.
The study regards uncertain quantities, e.g., survival times of mechanical components.

Desiderata:

→ Easy to interpret both for me as well as a decision maker
→ Fast algorithms for computing \( P(X = x | \epsilon) \)
→ A compact representation (wrt. # parameters)

Solution: (at least the one employed in this thesis)
Bayesian networks...
BN – An example

\[ P(G, A, S, T, C, E, U) \]
BN – An example

\[ P(G, A, S, T, C, E, U) \]

\[ \text{pa}(T) = \{A\} \]
BN – An example

- Gender ($G$)
- Age ($A$)
- Smoking ($S'$)
- Toxic ($T$)
- Cancer ($C$)
- Serum ($E$)
- Tumour ($U$)

Parents of $T$: $\text{pa}(T) = \{A\}$

Non-Parents of $T$: $\text{nd}(T) = \{A, S, G\}$

$$P(G, A, S, T, C, E, U)$$
BN – An example

\[ P(G, A, S, T, C, E, U) \]

\[ \text{pa}(T) = \{A\} \]
\[ \text{nd}(T) = \{A, S, G\} \]
\[ X \independent \text{nd}(X) \setminus \text{pa}(X) | \text{pa}(X) \]
\[ T \independent \{S, G\} | A \]
BN – An example

\[
\begin{align*}
T &\in (25, 65) & A \notin (25, 65) \\
T &\quad .05 & .01 \\
F &\quad .95 & .99
\end{align*}
\]

\[
P(T \mid A) = \begin{cases} 
  .05 & A \in (25, 65) \\
  .95 & A \notin (25, 65)
\end{cases}
\]

\[
\text{pa}(T) = \{A\}
\]

\[
\text{nd}(T) = \{A, S, G\}
\]

\[
X \perp \text{nd}(X) \setminus \text{pa}(X) \mid \text{pa}(X)
\]

\[
T \perp \{S, G\} \mid A
\]

\[
P(G, A, S, T, C, E, U) = P(G)P(A)P(S \mid G, A)P(T \mid A) \\
\quad \cdot P(C \mid S, T)P(E \mid C)P(U \mid C)
\]
Main ambition: To estimate the “quality” of maintenance.
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→ Four different root causes:
  - Deformation
  - Leakage
  - Breakage
  - Other mechanical

→ Failures classified as:
  - Critical failures
  - Degraded failures

→ Preventive maintenance:
  - Calendar based PM (8–12 months period)
  - Condition based PM
Paper I: Model

Repair-model: Imperfect repair – \( p \) and \( \pi \) – Brown & Proschan (1983)

PM-model: Random signs – \( q \) – Cooke (1996)

- \( \Xi^1_t = \) Effective age immediately after the \( t' \)th repair
- \( X^1_r = \) Potential time to critical failure for the \( r' \)th event
- \( Z^1_r = \) Potential time to PM for the \( r' \)th event
- \( Y^1_r = \) Time to event; \( Y^1_r = \min(Z^1_r, X^1_r) \)
- \( J^1_r = \) Indicator of event type; \( J^1_r = I(Z^1_r < X^1_r) \)
Paper I: Model

\[ X_r^1 \rightarrow Y_r^1 \rightarrow Z_r^1 \rightarrow J_r^1 \rightarrow Y^1 \rightarrow J^1 \rightarrow \Xi_r^1 \]

\[ \Xi_{r-1}^1 \rightarrow X_r^k \rightarrow Y_r^k \rightarrow Z_r^k \rightarrow J_r^k \rightarrow Y^k \rightarrow J^k \rightarrow \Xi_r^k \]

 ремонт-модель: несовершенное восстановление – Проблема и Проза (1983)


 \[ t = \text{Effective age immediately after the } t^{th} \text{ repair} \]

 \[ r = \text{Potential time to critical failure for the } r^{th} \text{ event} \]

 \[ Z_r^1 = \text{Potential time to PM for the } r^{th} \text{ event} \]

 \[ J_1^r = \text{Time to event}; J_r^1 = I(Z_r^1 < X_r^1) \]
Paper I: Results

A statistical model for competing risk and imperfect repair

→ “Quality of maintenance” can be measured by
  - $q$: The crew’s “eagerness”
  - $p$ and $\pi$: The crew’s “thoroughness”

→ *Naked failure-rate* easily accessible: Estimates from our model can be significantly different from the results of “standard techniques”

→ All parameters are identifiable
Paper II: Troubleshooting

→ The troubleshooting system is partly implemented in the commercial BATS system

→ Can be employed in many domains, initially intended for troubleshooting *printers*

→ We want to create a system that can find a “useful” strategy; *i.e.* a sequence of steps with a low *expected cost of repair*

→ The repair will be performed by untrained personnel: *Non-perfect* repair actions

→ Questions
Paper II: Model

```
\begin{center}
\begin{tikzpicture}
  \node (TOP) at (0,0) {TOP};
  \node (C1) at (-2,-1) {$C_1$};
  \node (C2) at (0,-1) {$C_2$};
  \node (C3) at (2,-1) {$C_3$};
  \node (C4) at (4,-1) {$C_4$};
  \node (X1) at (-3,-2) {$X_1$};
  \node (X2) at (-1,-2) {$X_2$};
  \node (X3) at (1,-2) {$X_3$};
  \node (X4) at (3,-2) {$X_4$};
  \node (X5) at (5,-2) {$X_5$};

  \draw[->] (TOP) -- (C1);
  \draw[->] (TOP) -- (C2);
  \draw[->] (TOP) -- (C3);
  \draw[->] (TOP) -- (C4);
  \draw[->] (C1) -- (X1);
  \draw[->] (C2) -- (X2);
  \draw[->] (C3) -- (X3);
  \draw[->] (C4) -- (X4);
  \draw[->] (X1) -- (X2);
  \draw[->] (X2) -- (X3);
  \draw[->] (X3) -- (X4);
  \draw[->] (X4) -- (X5);
\end{tikzpicture}
\end{center}
```
Paper II: Model

Diagram showing a model with multiple layers:
- **TOP**
- **System-layer**
  - \( C_1 \) to \( C_4 \)
  - \( X_1 \) to \( X_5 \)
- **Action-layer**
  - \( A_1 \) to \( A_5 \)

Legend:
- \( A \): Action
- \( C \): Condition
- \( R \): Result
- \( Q \): Query (Question-layer)
Paper II: Model

system-layer

action-layer

result-layer
Paper II: Results

→ A troubleshooting model of coherent systems to easily identify a “useful” repair strategy

→ Considerably improvement over the greedy approach in many cases

→ Careful modelling of user interactions:
  • Non-perfect repair-actions
  • Questions

→ The BN representation allows a fast calculation scheme
Paper III: Classification

→ From a set of attributes describing an object we want to classify this object as a member of one class (out of a predefined set of classes)

→ Classification learning (supervised pattern recognition) is to automatically learn a classifier from a labelled database

→ Data mining: Both # attributes and # training cases are large. Computational complexity becomes important
Paper III: Model

Class: Male or Female?

Naïve Bayes
Duda&Hart (1973)

\[ C \]

\[ A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \quad A_6 \]

\[ A_1 \quad \text{Often seen at Lerkendal?} \quad A_2 \quad \text{“Busy” Saturdays?} \]
\[ A_3 \quad \text{Has Liverpool-shirt?} \quad A_4 \quad \text{Football fan?} \]
\[ A_5 \quad \text{Responsible for grocery shopping?} \quad A_6 \quad \text{Cooks dinner?} \]
Paper III: Model

Class: Male or Female?

Tree Augmented Naïve Bayes
Friedman et al. (1997)

$A_1$ Often seen at Lerkendal?  $A_2$ “Busy” Saturdays?
$A_3$ Has Liverpool-shirt?  $A_4$ Football fan?
$A_5$ Responsible for grocery shopping?  $A_6$ Cooks dinner?
Paper III: Model

Class: Male or Female?

Hierarchical Naïve Bayes
Zhang et al. (2002) &
Paper III

$L_1$ Football?
$L_2$ Housekeeper?

$A_1$ Often seen at Lerkendal?
$A_2$ “Busy” Saturdays?
$A_3$ Has Liverpool-shirt?
$A_4$ Football fan?
$A_5$ Responsible for grocery shopping?
$A_6$ Cooks dinner?
Relative difference in accuracy, our system compared with Naïve Bayes, Tree Augmented Naïve Bayes, C5.0, and Neural Networks.
Papers IV&V: The OOBN framework

→ *Object oriented BNs* (Bangsø & Wuillemin, 2000) is a framework for handling *large* domains

→ Focus is on:
  - Repetitive substructures — Instantiations of a class
  - “Similar” structures — Class hierarchies

Bayesian network
Papers IV & V: The OOBN framework

→ *Object oriented BNs* (Bangsø & Wuillemin, 2000) is a framework for handling *large* domains

→ Focus is on:
  - Repetitive substructures — Instantiations of a class
  - “Similar” structures — Class hierarchies
Paper IV: Parameter learning in OOBNs

→ A method for estimating parameters in OOBNs is required

Learn in the class specifications

→ Potential problems:

- OO learning when the domain is not “entirely” OO
- Not fully specified OO model – Type uncertainty

Bayesian network

Class library

\[ \mathcal{D}, \hat{\theta} \]
Paper IV: Results

- Conventional
- OMD network - Type uncertainty, 25% missing data

a) Fully specified model

b) Type uncertainty
Paper V: Structural learning in OOBNs

Structural learning:

Find the graph that best fits the data.

→ Can calculate the marginal likelihood of a graph by local computations in BNs (Cooper&Herskovits, 1991).

→ Combined with a prior we can find the posterior distribution over the model space

This can be generalized to OOBNs:

→ OOBN information can be easily encoded and utilized

→ Learning algorithm is consistent

→ Type uncertainty through discrete optimization
Paper V: Results

a) Insurance domain

b) Type uncertainty
Summary

→ Bayesian Networks . . .
→ . . . with Applications in Reliability Analysis
  ● Maintenance model
  ● Fault-finding system
  ● Classification (e.g., of components requiring extensive maintenance)
  ● Learning in a framework for large domains (e.g., to perform an availability study)