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## Lexical analysis: Deterministic Automata

## What we have

- A file, when you read it, is just a sequence of numbers from 0 to 255 (bytes):
$72,101,108,108,111,32,119,111,114,108,100, \ldots$
- By convention, some of them stand for letters and numbers:
'H', 'e', 'r', 'r, 'o', ' ', 'w','o', r', ', ', 'd', ...
- At this level, a source program just looks like a gigantic pile of bytes, which is not very informative


## What we don't want

- A programming language key word like, say, "while" will appear as the sequence
w (119), h (104), i (105), I (108), e (10)
and it would be very tiresome to write a compiler that detects this sequence every time the programmer wants to start a while loop.
- You can't stop them from calling a variable 'whilf':
$\mathbf{w}$ (119), $\mathbf{h}(104), \mathbf{i}(105), \mathbf{I}(108)$, (looks like we're starting a loop soon...)
...f(102) (dang, rewind to 119 and try again, this is not a loop)


## What we want

- A neat and tidy grouping of characters into meaningful lumps, so that we can operate on those without caring about the characters they are made up from:

is easier to read as
if (whilf == 2 ) $\{x=5 ;\}$
because characters are grouped together as words and punctuation.
- We could even make the color-coding meaningful:
keywords and punctuation
delimiters of groups
variables
operators
numbers


## What are the colors for?

- Consider this statement we already looked at:

$$
\text { if ( whilf }==2 \text { ) }\{x=5 ;\}
$$

- Consider this statement also:
while ( $\mathrm{a}<42$ ) $\{\mathrm{a}+=2 ;\}$
if we respect the same coloring, it piles up as
while ( $\mathrm{a}<42$ ) $\{\mathrm{a}+=2 ;$ \}
- These two statements have wildly different meanings, but they share the same structure as far as our colors are concerned:
blue red green purple yellow red red green purple yellow blue red
- The structure they share is syntactic (or grammatical, if you like)
- The difference between them is lexical
- We're talking about lexical analysis today, but we'll need both, so we'll (eventually) try to get both from the stream of meaningless data.


## Three useful words

- Lexeme
- Lexemes are units of lexical analysis, words
- They're like entries in the dictionary, "house", "walk", "smooth"
- Token
- Tokens are units of syntactical analysis,
- They are units of sentence analysis, "noun", "verb", "adjective"
- Semantic
- This is what something means, there is no sensible unit
- It's like explanations in the dictionary
- "house: a building which someone inhabits"
- "walk: the act of putting one foot in front of the other"
- "smooth: the property of a surface which offers little resistance"


## Classes of lexemes

- Some of the words we want to classify are fixed:
- "if"
- "while"
- "for"
- "=="
...et cetera...
- Other classes have countably infinite instances:
$-1$
$-2$
- ...
- ...65536...

These are all specific cases of "integer"

## Finite Automata

- We need a mechanism to identify not just single, specific words, but entire classes of them
- Forget all about specific numbers for a while, let's just try to find out whether we can make a rule to recognize a number when we see one
- Here's a deterministic finite automaton, (drawn as a directed graph, because that's easy to follow):

(You may remember these things from discrete mathematics, but l'll repeat them anyway)


## Anatomy of a DFA

The edges/arcs represent transitions between states


These are the states (1, 2 and 3 )

## Start and finish

- One state is singled out as the starting state
- One or more states are identified as accepting states
- I've colored them green here, other common notations are to use a double circle or thicker lines
- Doesn't matter as long as we can tell what it means



## Labels on the arcs

- Transitions are marked with sets of single characters that they apply to
- '.' means the period character




## Traversing the graph

- The idea is that we start by pointing a finger at the starting state, and then
- Read a character of text
- Search for any transitions which are labeled with that character
- Throw away* the character, and point at the new state instead
- Repeat with another character until something fails
- When something fails, we're either pointing at an accepting state, or not.
- If we are, the automaton accepts the text we read
- If we are not, the text was wrong**
* Programs won't actually discard it, but the finite automaton no longer cares what it was
** "wrong" isn't really the best word, but it'll do for now


## Take "42.64"

- We start in state 1
- Read '4’
- Find a transition



## We're left with "2.64"

- We're in state 2
- Read '2'
- Find a transition



## We're left with ".64"

- We're in state 2
- Read '.'
- Find a transition



## We're left with " 64 "

- We're in state 3
- Read ' 6 '
- Find a transition



## We're left with "4"

- We're in state 3
- Read '4'
- Find a transition



## We're out of characters...

- ...and standing in state 3
- That's an accepting state, so this automaton recognizes the word " 42.64 "
- The state sequence (1,2,2,3,3,3) which we just constructed is a proof of that
(it's not so important to call this "a proof", but a couple of other proofs in this subject are constructed by just following a recipe, so we might as well say it right away.)

$$
[0-9] \quad[0-9]
$$



## That was one class of words

- The DFA we just looked at recognizes integers with an optional (possibly empty) fractional part
- How would you change it to reject, say, "42." while still accepting "42.0", or accept ". 64"?
- Discriminating between all the classes of words in an entire programming language requires a whole bunch of different DFAs to work in conjunction
- Luckily, we can program them very generally


## An alternative view

- One of the neat things about graphs is that we can write them up as tables
- Consider:



## Here's "42.64" again, in the table view

- State 1 , read '4’, go to state 2

| State | $[0-9]$ | $\because$ | $<$ other | Accept? |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | - | No |
| 2 | 2 | 3 | - | Yes |
| 3 | 3 | - | - | Yes |

- State 2, read '2', go to state 2

| State | $[0-9]$ | $\because \prime$ | $<o t h e r>$ | Accept? |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | - | No |
| 2 | 2 | 3 | - | Yes |
| 3 | 3 | - | - | Yes |

## Here's "42.64" again, in the table view

- State 2, read '.', go to state 3

| State | $[0-9]$ | $\because$ | $<$ other | Accept? |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | - | No |
| 2 | 2 | 3 | - | Yes |
| 3 | 3 | - | - | Yes |

- State 3, read '6’, go to state 3

| State | $[0-9]$ | $\because$ | $<$ other $>$ | Accept? |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | - | No |
| 2 | 2 | 3 | - | Yes |
| 3 | 3 | - | - | Yes |

## Here's "42.64" again, in the table view

- State 3, read '4’, go to state 3

| State | $[0-9]$ | $\because \prime$ | $<$ other | Accept? |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | - | No |
| 2 | 2 | 3 | - | Yes |
| 3 | 3 | - | - | Yes |

- State 3, out of input, accept

| State | $[0-9]$ | $\because \prime$ | $<$ other | Accept? |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | - | No |
| 2 | 2 | 3 | - | Yes |
| 3 | 3 | - | - | Yes |

## Implementation

- This is the algorithm in Dragon Fig. 3.27, p. 151
- Store state (it's just a row index into the table)
- Read character (it's just a column index)
- Set state to the new one in the table
- Repeat
- The beauty of this is that the same program logic works for any DFA, changes in the automaton only require a different table to work with, not a different algorithm


## So far, so good

- We have a graph representation that we can draw on paper and follow by pointing fingers at the graph and text
- We have a table representation that we can turn into a program


## Where we are going with this

- Programming a word-class recognizer (lexical analyzer, or scanner) with ad-hoc logic is complicated and error-prone
- Writing one using tables is a little easier, but requires punching in a bunch of boring table entries to represent specific DFAs
- Generating one is very convenient:
- Specify word classes as regular expressions
- Let a program write a gigantic table of states that includes all of the expressions


## How can such a generator work?

- We'll need to write down the graph differently, programs have a really hard time understanding pictures
- We'll need a path from that notation and into tables
- Doing it automatically will give us bigger tables than we need
- and thus, a great opportunity to shrink them to a minimum
(Stick around for the mesmerizing sequel, "Lexical Analysis II: Attack of the NFA")

