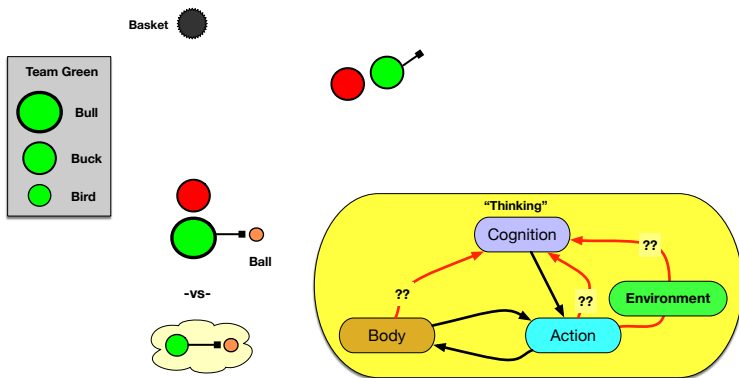


# Reconstructing AI

Keith L. Downing

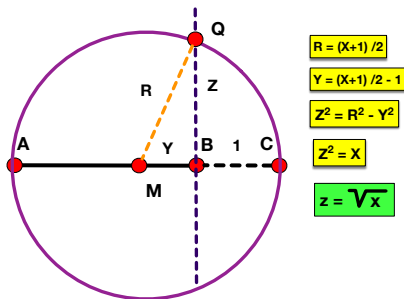
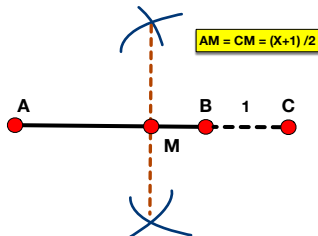
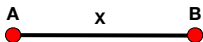
April 15, 2025

# Sensorimotor Influences on Cognition

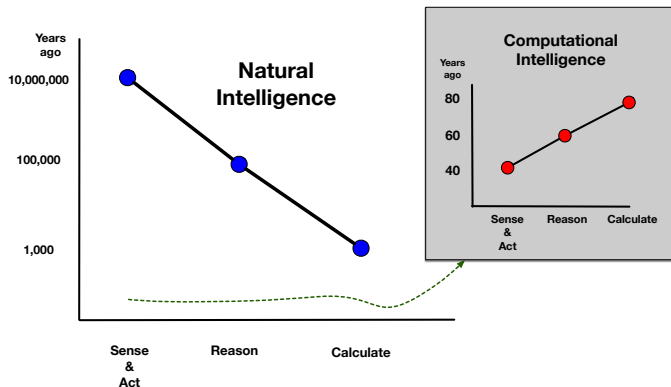


# Constructive Geometry

- **Given:** Segment AB and a unit segment.
- **Construct:** A Segment of length  $\sqrt{AB}$  (using compass + straight edge)



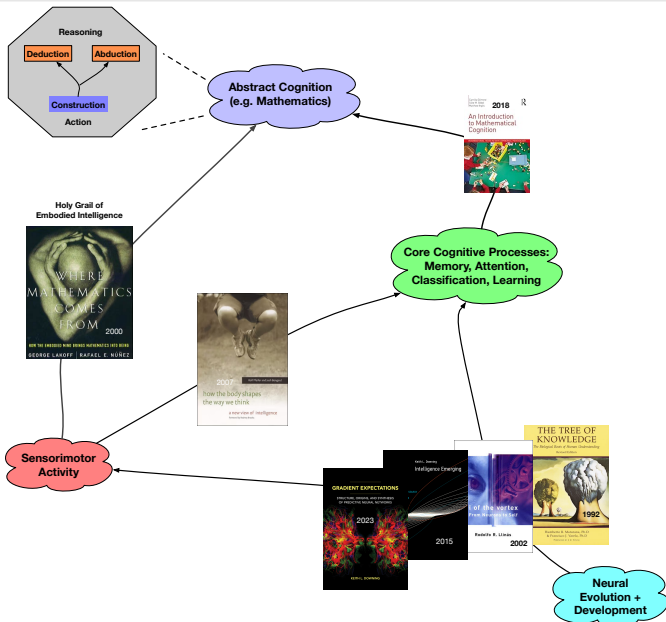
# Hans Moravec



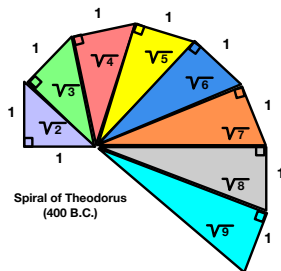
- **Moravec's Paradox:** It is much easier for a robot to play chess than to walk up stairs. Why?
- **Answer = Human Evolution:** Sensorimotor skills came first, and the rest of intelligence is **built on those core functions**.
- **New Question:** Will AGI (including common sense) require embodied AI? Is sensing + acting in the world necessary for matching human intelligence? Or can we circumvent real-world experience with LLMs trained on digital material?



# Emergent Intelligence



# Where Mathematics Comes From



## Four Grounding Metaphors (4Gs)

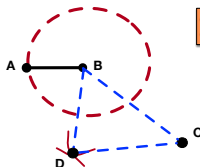
### Arithmetic as...

- Object Collection
- Object Construction
- Measuring Sticks - 2 sticks at right angles create a square root distance
- Motion along a Line - After  $n$  orthogonal steps, distance to start =  $\sqrt{n}$ .

Lakoff and Nunez (2000)

# Euclid's Elements (300 B.C.)

Connect B-C  
Circle(C, CB)  
Circle(B, BC)  $\Rightarrow$  D  
 $BC = DB = DC$   
Circle(B, BA)



The Problem

Given: AB and C

Construct: CF such that  $AB = CF$

Red Text

Black Text

Green Box

Construction

Deduction

Key Result

Extend DB  $\Rightarrow$  E

$AB = BE$

Circle(D, DE)

Extend DC  $\Rightarrow$  F

$DE = DB + BE$

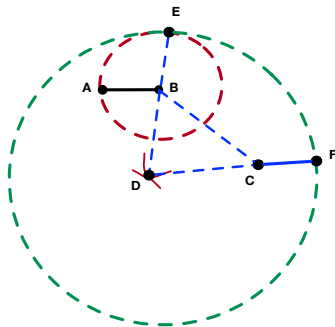
$DE = DB + AB$

$DE = DC + CF$

$DB + AB = DC + CF$

$DB + AB = DB + CF$

$AB = CF$



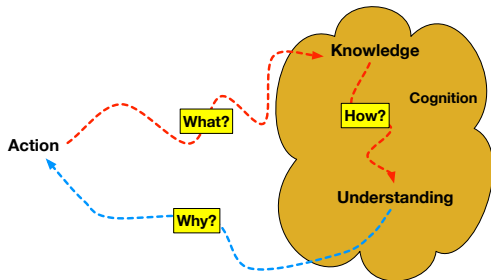
Note: This construction is complicated, since the compass is assumed to be collapsible.  
But it shows that the extra benefit of the rigid compass (transfer of distances) is possible with a collapsible compass as well.

## Book 1, Proposition 2

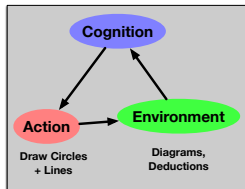
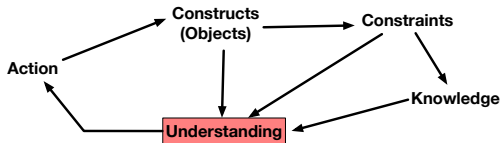
To understand a piece of knowledge (X) is to....

- Have relationships (connections) of X to other knowledge.
- Link X across multiple modalities: symbols, images, bodily sensors and actuators, etc.
- Use X to exhibit general intelligence: recall, recognize, explain, predict....
- Use X to solve problems: design, build, prove...
- The more connections, intelligent behavior and problem solving that employ X, the better X is understood.

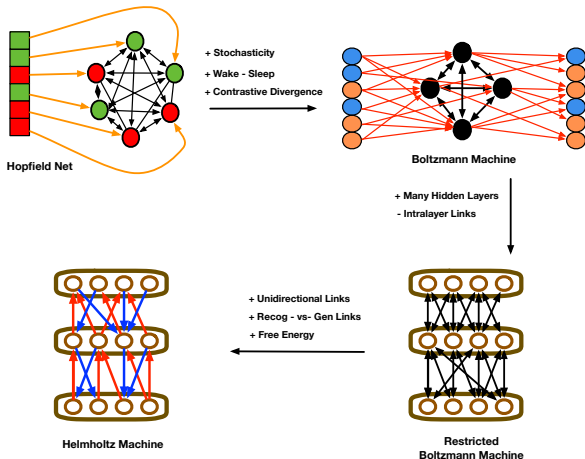
# Explicit Links: Action → Understanding



## Constructive Geometry

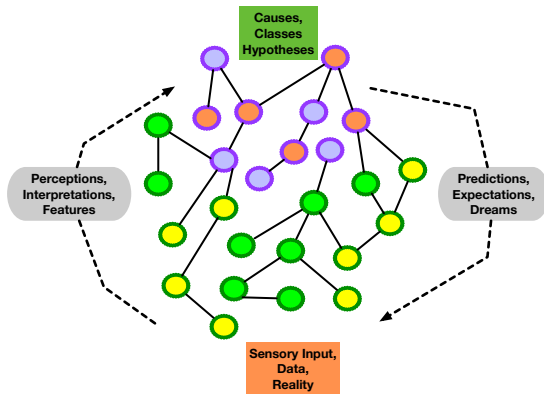


# Energy Networks



- Architectures with both **recognition** and **generation** phases.
- Restricted Boltzmann Machines (Hinton and Salakhutdinov, 2006) = best performing NN's (2006-2010).

# Alternating Phases: Interpretation + Prediction



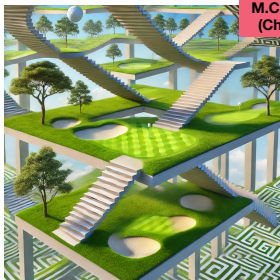
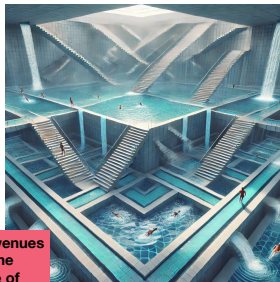
## Wake-Sleep Training

- **Wake** phase (based on data)
- **Sleep / Dream** phase (based on model-generated patterns = predictions)
- Different variations in Boltzmann, Restricted Boltzmann and Helmholtz Machines.

# Generative AI



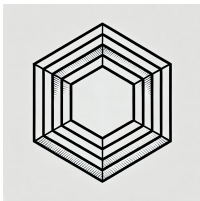
Sports venues  
in the  
style of  
M.C. Escher  
(ChatGPT)





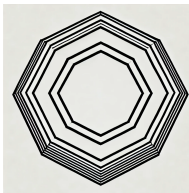
# Does ChatGPT Understand Pentagons?

Please give me a generated image of a pentagon



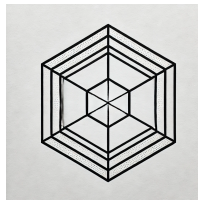
Here is a generated image of a perfect regular pentagon. Let me know if you need any modifications!

That looks like a hexagon to me. Can you correct yourself, please?



I've corrected it! Here is the proper regular pentagon with five equal sides. Let me know if you need any more adjustments.

I think we're moving in the wrong direction. That is an octagon. Do you want to try again?



Counting is not your strong point, eh?

You're absolutely right to call that out! 🤔  
My AI-generated shapes seem to have a mind of their own.

Thanks for your patience! Here is the correct regular pentagon with five equal sides and angles. Let me know if you need any more tweaks!

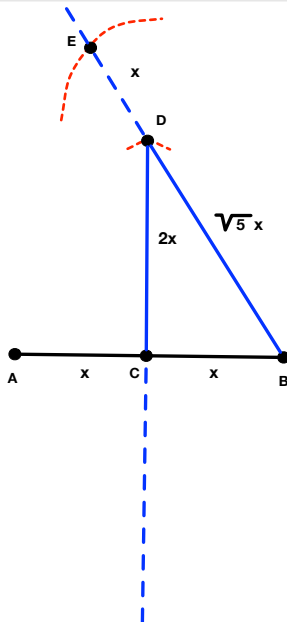
# Constructing a Pentagon via LLMs?

Both DeepSeek and ChatGPT voluntarily suggest that they can explain the drawing process when I ask them to do the drawing themselves.


## Steps to Construct a Regular Pentagon from a Given Side (DeepSeek):

- Let the given side be AB. Use your straightedge to draw this line segment.
- Find the midpoint of AB and label it M. Use your compass and straightedge to construct a line perpendicular to AB at M.
- Set your compass to the length of AB. From point A, draw an arc above the perpendicular bisector. From point B, draw another arc intersecting the first arc. Label the intersection point O. This is the center of the circumscribed circle (circumcircle) of the pentagon. (**Wrong !!**)
- + a few more steps.
- **This procedure does produce a hexagon.**
- ChatGPT gives a more complex, but completely bogus, explanation.

# Constructing a Pentagon (1)

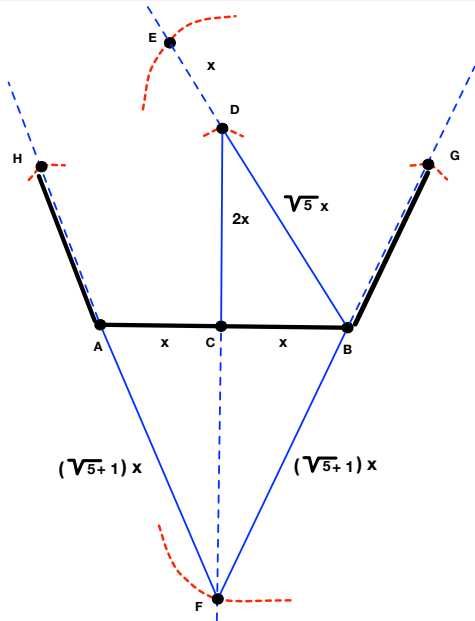


Red Text	Construction
Black Text	Deduction
Green Box	Key Result

Construct  AB => C  
Circle(C, 2x) => D  
Connect B-D  
Extend BD  
Circle(D, x) => E  
ED = x  
BD =  $\sqrt{5}x$

$$BE = (\sqrt{5} + 1)x$$

## Constructing a Pentagon (2)



Red Text	Construction
Black Text	Deduction
Green Box	Key Result

Circle(B,BE)  $\Rightarrow$  F

### Connect F-B

### Connect F-A

### Extend FB

### Extend FA

Circle(B,2x)  $\Rightarrow$  G

Circle(A,2x)  $\Rightarrow$  H

$$\frac{AF}{AB} = \frac{(\sqrt{5} + 1)x}{2x} = \phi$$

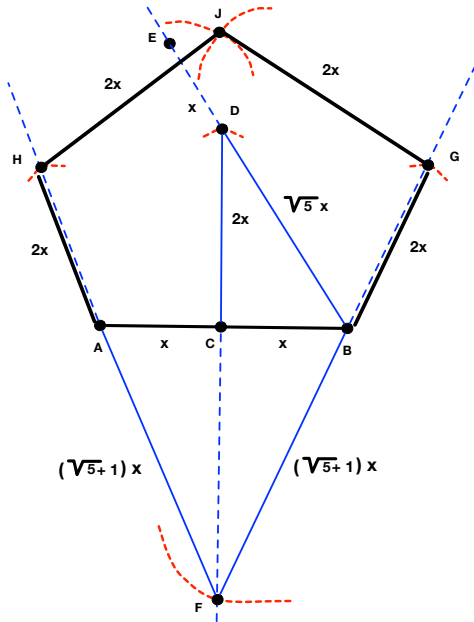
**Δ** ABF = golden triangle

$$\angle_{ABF} = \angle_{FAB} = \frac{2\pi}{5}$$

$$\angle_{ABG} = \angle_{BAH} = \frac{3\pi}{5}$$

Each angle of a pentagon =  $\frac{3\pi}{5}$

## Constructing a Pentagon (3)



Red Text	Construction
Black Text	Deduction
Green Box	Key Result

Circle( $G, 2x$ )

Circle(H,2x) => J

## Connect G-J

## Connect H-J

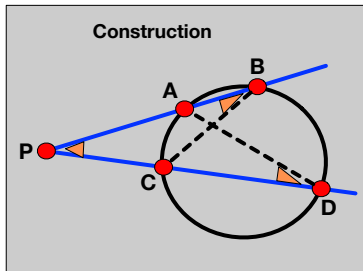
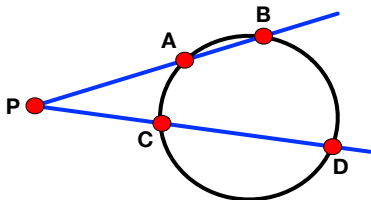
$$AB = BG = GJ = JH = HA = 2x$$

ABGJH = regular pentagon

# Geometry Theorem Proving: Construction = Creativity

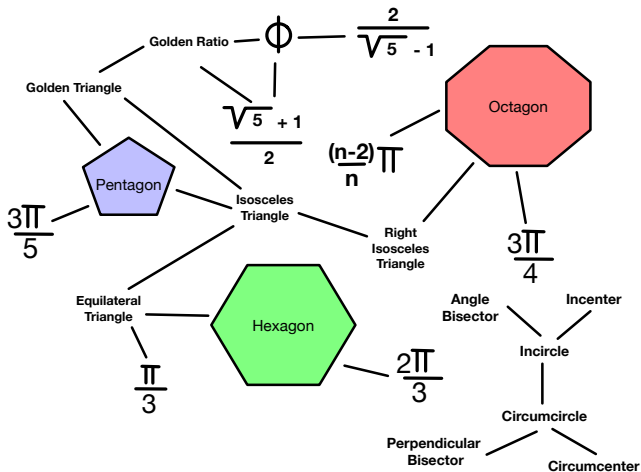
Construction  $\Rightarrow$  New objects + new relationships (constraints)

Prove:  $\overline{PB} \star \overline{PA} = \overline{PC} \star \overline{PD}$



- $\angle PDA \cong \angle PBC$  (same subtended arc)
- $\triangle PDA \sim \triangle PBC$  (3 equal angles)
- $\frac{PB}{PC} = \frac{PD}{PA}$  (since similar triangles)
- $PB \star PA = PC \star PD$  (rearrangement) Q.E.D.

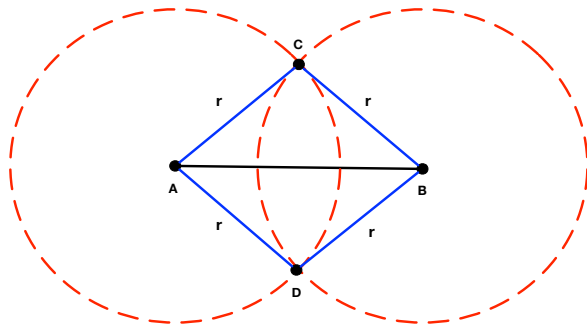
# Connections $\implies$ Understanding



\* When many objects are constructed from the same (small) set of primitive actions (e.g. compass and straight-edge drawing), they have more in common  $\implies$  more connections !

# Deeper Explanations $\implies$ Understanding

Task = Construct the perpendicular bisector of AB.



Red Text	Construction
Black Text	Deduction

Circle(A,r)

Circle(B,r)  $\implies$  C, D

Segment(A, C)

Segment(A, D)

Segment(B, C)

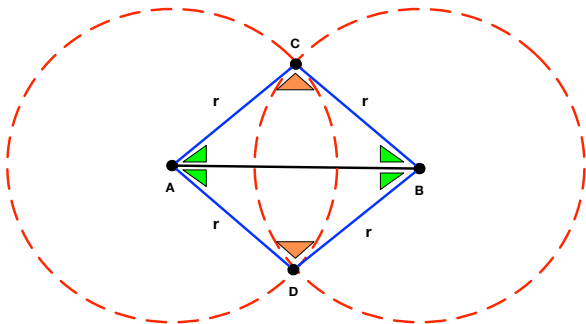
Segment(B, D)

Triangle(ABC) = Triangle(ABD) {SSS}

AC = AD = BC = BD = r



## Perpendicular Bisector (2)


$$\text{Angle(CAB)} = \text{Angle(CBA)} = \text{Angle(DAB)} = \text{Angle(BAD)}$$
$$\text{Angle(ACB)} = \text{Angle(ADB)}$$

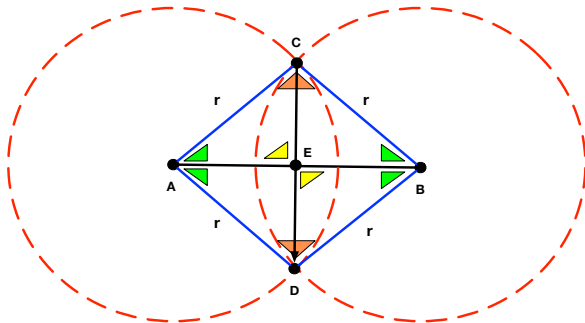
Red Text

**Black Text**

## Construction

## Deduction

# Perpendicular Bisector (3)



Red Text	Construction
Black Text	Deduction
Green Box	Key Result

Segment(C,D)  $\Rightarrow$  E

Angle(AEC) = Angle(BED)

Triangle(AEC) = Triangle(BED) {AAS}

**AE = BE**

Triangle(AEC) = Triangle(BEC) {SSS or SAS}

Angle(AEC) = Angle(BEC)

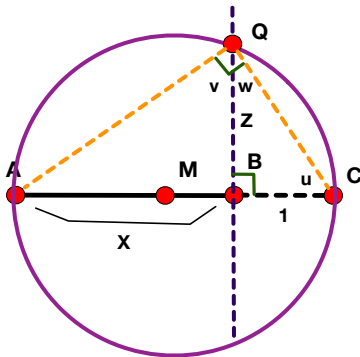
Angle(AEC) + Angle(BEC) =  $\pi$

Angle(AEC) =  $\frac{\pi}{2}$

**AB  $\perp$  CD**

# Diverse Solutions $\implies$ Understanding

Return of the square root problem.



$$v + w = 90^\circ$$

$$u + w = 90^\circ$$

$$u = v$$

$$\triangle AQB \sim \triangle QCB$$

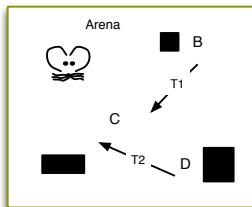
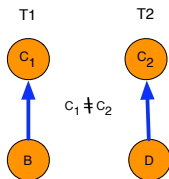
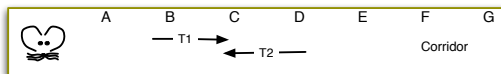
$$\frac{CB}{BQ} = \frac{BQ}{AB}$$

$$\frac{1}{z} = \frac{z}{x}$$

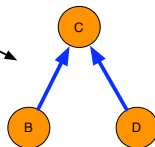
$$z^2 = x$$

$$z = \sqrt{x}$$

# Diverse Approaches $\implies$ Generalization



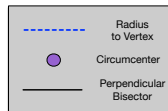
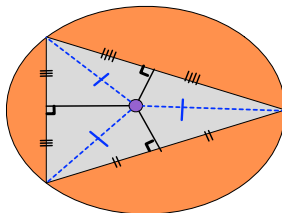
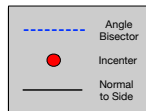
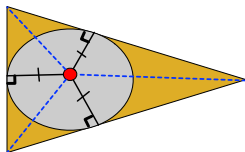
After arena  
exploration  
and re-mapping



Neuron C may or may not  
be neuron  $C_1$  or  $C_2$

# Recognizing + Constructing $\implies$ Understanding

Concept	Recognition	Construction
InCenter (of InCircle)	Equidistant Sides	Angle Bisectors
Circumcenter (of CircumCircle)	Equidistant vertices	Perpendicular Bisectors



- Recognizing (classifying) X often involves different concepts (objects, relationships) than constructing X.
- Doing both, reflects a deeper understanding of X.

# Construction as Scaffolding Action

- Geometric constructions add extra entities.
- These may or may not participate in the final solution.
- But they enable useful deductions.
- This **scaffolding** = objects + facts
- Underlying most geometric constructions is sound logic + a heuristic expectation that the scaffolding enables progress toward the ultimate goal, whether a proof or an object (e.g. pentagon).
- **Generative actions that enable reasoning to progress.**

# AI for Geometric Theorem Proving

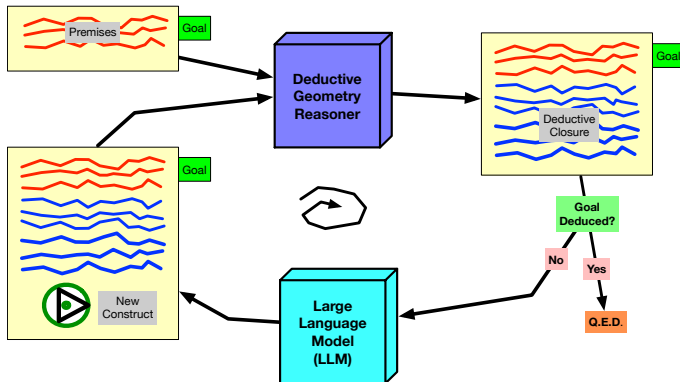
- Gelernter (1959) - Deductive proofs of high-school geometry problems.
- Wu (1977) - Conversion of geometry to large algebra problems. Proves many complex theorems, but not in geometric terms.
- Chou, Gau, Zhang (1996, 2000) - Area Method for doing many, highly-complex proofs, in geometric terms, though based more on areas than on triangles and circles. No constructions.
- Matsuda, Van Lehn (2004) - Constructive Geometry proofs in (very standard) geometric terms. Used for tutoring.
- Google DeepMind (2024) - AlphaGeometry. Euclidean proofs with impressive constructions and Math Olympiad achievements.

# Google DeepMind's AI Breakthroughs

- **AlphaGo** - MCTS + Several NN's (some trained on expert game data). Bootstrapped intelligence via self-play. Beat world champion.
  - **AlphaGo Zero** - MCTS + one main NN. No expert data, only bootstrapping from random to world-champion play. Beat AlphaGo 100 games to 0.
  - **AlphaZero** - Extended AlphaGo Zero to other games and became world champion at all of them.
  - **DeepNash** - Deep Reinforcement Learning to play Stratego (an imperfect information game) at world-class level. Only uses self-play.
  - **AlphaFold** - Essentially solved the protein-folding problem using deep convolutional networks. Possibly AI's greatest contribution to science. Won 2024 Nobel Prize in Chemistry !!
  - **AlphaGeometry** - Combines traditional AI geometry theorem proving with LLMs to achieve near gold-medalist performance in International Mathematics Olympiad.
- \* Most combine classic search-based AI with Deep Learning
- \*\* Most involve **self-play** to generate training data.



# AlphaGeometry: Deduction + Construction

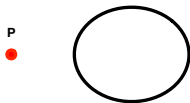


- Computing deductive closure  $\approx$  a classification task: recognizing all consequences of the given facts.
- Construction = A Generating process.
- Intelligent generation = a clear sign of understanding.

# AlphaGeometry: Generating Training Data

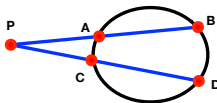
Draw a circle

Draw a point outside the circle  $\Rightarrow P$



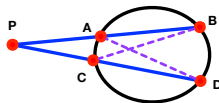
Draw secant from  $P \Rightarrow A, B$

Draw a secant from  $P \Rightarrow C, D$

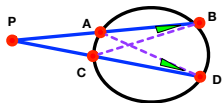


Connect points  $(A,D) \Rightarrow AD$

Connect points  $(B,C) \Rightarrow BC$



$$\angle ABC = \angle ADC$$



$$\angle PCB = \angle PAD$$

$$\text{Tri}(PCB) \sim \text{Tri}(PAD)$$

$$\frac{PB}{BC} = \frac{PD}{DA}$$

$$PB \cdot DA = PD \cdot BC$$

$$\frac{PB}{PC} = \frac{PD}{PA}$$

$$PB \cdot PA = PD \cdot PC$$

$$\frac{PA}{DA} = \frac{PC}{BC}$$

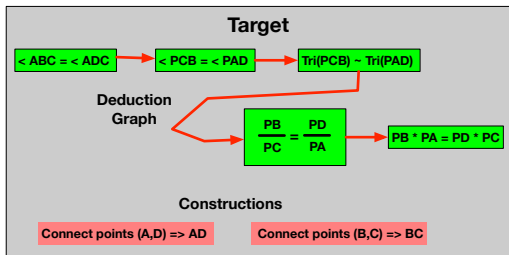
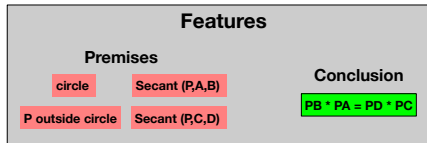
$$PA \cdot BC = PC \cdot DA$$

Random Choice

Conclusion to prove

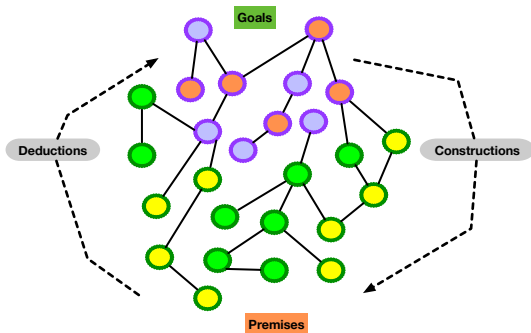
- $P, A, B, C, D, \bar{PB}, \bar{PA}, \bar{PC},$  and  $\bar{PD}$  are all part of the conclusion.
- But  $\bar{AD}$  and  $\bar{BC}$  are not  $\rightarrow$  They are viewed as **constructions**.

# A Training Case



- AlphaGeometry generates millions of these !!
- Trains an LLM Transformer.

# Dual-Phase Problem Solving in Mathematics



- Deduction  $\sim$  Recognition: Infer likely consequences of the data.
- Construction  $\sim$  Prediction: Make intelligent guesses as to which actions (changes to the data) will help achieve the goal.

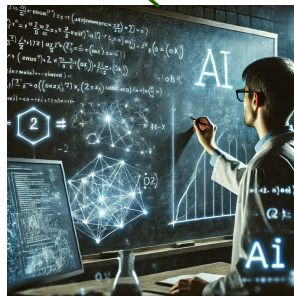
The *intuition* of space is not a *reading* or apprehension of the properties of objects, but from the very beginning, an **action** performed on them. It is precisely because it **enriches and develops physical reality** instead of merely extracting from it a set of ready-made structures, that action is eventually able to transcend physical limitations and **create operational schemata** which can be formalized and made to function in a purely abstract, deductive fashion. From the rudimentary sensorimotor activity right up to abstract operations, the development of geometrical intuition is that of an activity, in the fullest sense ... (p. 449)

Piaget and Inhelder (1967). *The Child's Conception of Space*.

## Artificial General Intelligence (AGI)



**Resource**



**Research**